EE 508 Lecture 17

Basic Biquadratic Active Filters

Second-order Bandpass
Second-order Lowpass
Effects of Op Amp on Filter Performance

Review from Last Time

Comparison of Transforms

LP to BP

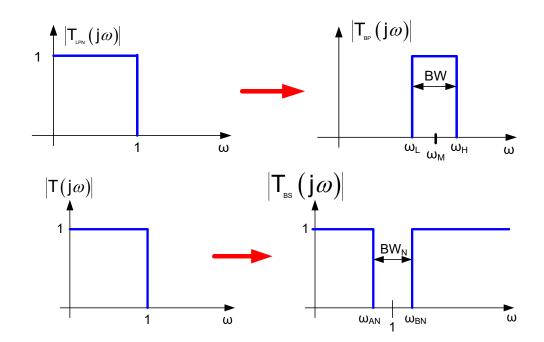
$$s \rightarrow \frac{s^2 + 1}{s \cdot BW_M}$$

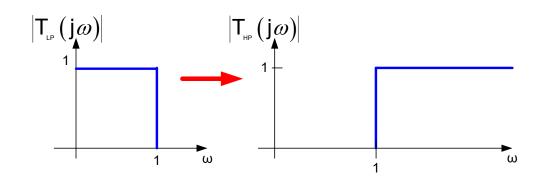
LP to BS

$$s \rightarrow \frac{s \bullet BW_N}{s^2 + 1}$$

LP to HP

$$s \rightarrow \frac{1}{s}$$





Filter Design Process

Establish Specifications

- possibly $T_D(s)$ or $H_D(z)$
- magnitude and phase characteristics or restrictions
- time domain requirements

Approximation

- obtain acceptable transfer functions $T_A(s)$ or $H_A(z)$
- possibly acceptable realizable time-domain responses

Synthesis

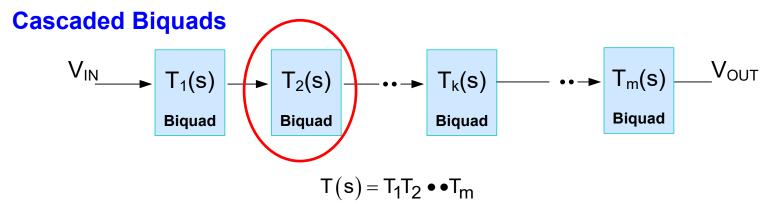
- build circuit or implement algorithm that has response close to $T_A(s)$ or $H_A(z)$
- actually realize T_R(s) or H_R(z)



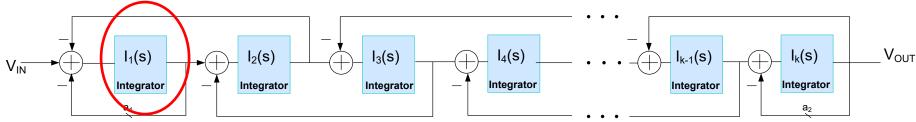
There are many different filter architectures that can realize a given transfer function

Considerable effort has been focused over the years on "inventing" these architectures and on determining which is best suited for a given application

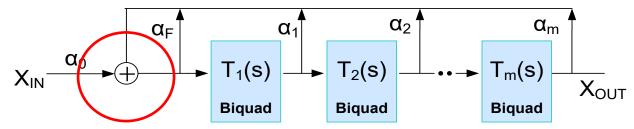
Most even-ordered designs today use one of the following three basic architectures





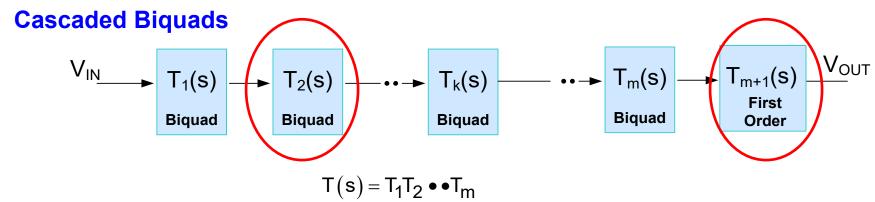


Multiple-loop Feedback (less popular)

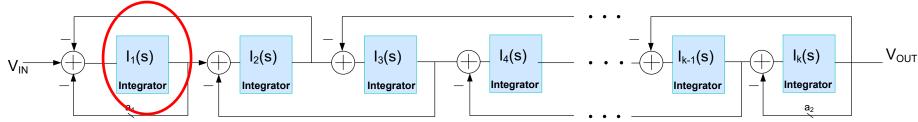


What's unique in all of these approaches?

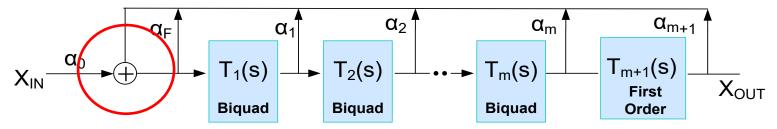
Most odd-ordered designs today use one of the following three basic architectures





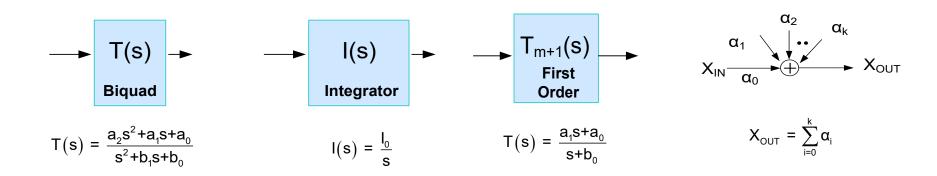


Multiple-loop Feedback (less popular)



What's unique in all of these approaches?

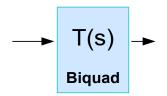
What's unique in all of these approaches?



- Most effort on synthesis can focus on synthesizing these four blocks (the summing function is often incorporated into the Biquad or Integrator)
 (the first-order block is much less challenging to design than the biquad)
- Some issues associated with their interconnections
- And, in many integrated structures, the biquads are made with integrators (thus, much filter design work simply focuses on the design of integrators)

Biquads

How many biquad filter functions are there?



$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$$

$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$$
 $a_0 \neq 0, a_1 \neq 0, a_2 \neq 0$

$$a_0 \neq 0, \ a_1 \neq 0, \ a_2 \neq 0$$

$$T(s) = \frac{a_0}{s^2 + b_1 s + b_0}$$

$$a_0 \neq 0$$

$$T(s) = \frac{a_2 s^2 + a_0}{s^2 + b_1 s + b_0} \qquad a_0 \neq 0, \ a_2 \neq 0$$

$$a_0 \neq 0, a_2 \neq 0$$

$$T(s) = \frac{a_1 s}{s^2 + b_1 s + b_0}$$

$$a_1 \neq 0$$

$$T(s) = \frac{a_1 s + a_0}{s^2 + b_1 s + b_0}$$

$$a_0 \neq 0, a_1 \neq 0$$

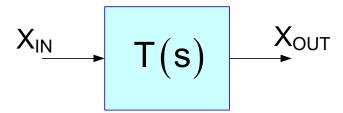
$$T(s) = \frac{a_2 s^2}{s^2 + b_1 s + b_0}$$

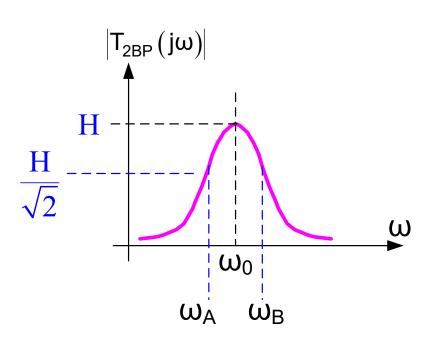
$$a_2 \neq 0$$

$$T(s) = \frac{a_2 s^2 + a_1 s}{s^2 + b_4 s + b_6}$$

$$a_2 \neq 0, \ a_1 \neq 0$$

Review: Second-order bandpass transfer function





$$\begin{aligned} \left| \mathsf{T}_{\mathsf{2BP}} \left(\mathsf{s} \right) \right| &= \mathsf{H} \frac{\mathsf{s} \left(\frac{\omega_0}{\mathsf{Q}} \right)}{\mathsf{s}^2 + \mathsf{s} \left(\frac{\omega_0}{\mathsf{Q}} \right) + \omega_0^2} \\ \mathsf{BW} &= \omega_{\mathsf{B}} \text{-} \omega_{\mathsf{A}} = \frac{\omega_0}{\mathsf{Q}} \\ \omega_{\mathsf{PEAK}} &= \omega_0 \\ \omega_0 &= \sqrt{\omega_{\mathsf{A}} \omega_{\mathsf{B}}} \end{aligned}$$

There are many different filter architectures that can realize a given transfer function

Will first consider second-order Bandpass filter structures

$$T(s) = H \frac{s\left(\frac{\omega_0}{Q}\right)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

$$BW = \omega_B - \omega_A = \frac{\omega_0}{Q}$$

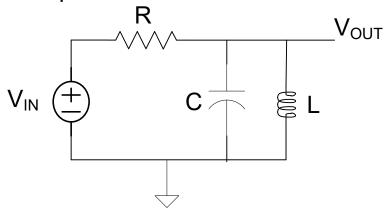
$$\omega_{PEAK} = \omega_0$$

$$\omega_0 = \sqrt{\omega_A \omega_B}$$

There are many different filter architectures that can realize a given transfer function

Will first consider second-order Bandpass filter structures

Example 1:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{RC} \frac{s}{s^2 + s \left(\frac{1}{RC}\right) + \frac{1}{LC}}$$

Second-order Bandpass Filter

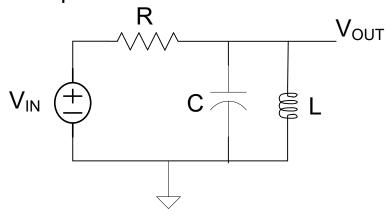
- 3 degrees of freedom
- 2 degrees of freedom for determining dimensionless transfer function (impedance values scale)

$$\omega_0 = ?$$

$$Q = ?$$

$$BW = ?$$

Example 1:



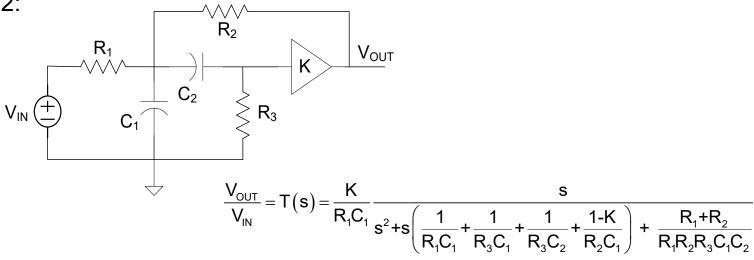
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{RC} \frac{s}{s^2 + s \left(\frac{1}{RC}\right) + \frac{1}{LC}}$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$
 $Q = R\sqrt{\frac{C}{L}}$ $BW = \frac{1}{RC}$

Can realize an arbitrary stable 2^{nd} order bandpass function within a gain factor Simple design process (sequential but not independent control of ω_0 and Q) If trimming is necessary, prefer to trim with a single resistor

Can't trim this filter with single resistor

Example 2:



Second-order Bandpass Filter

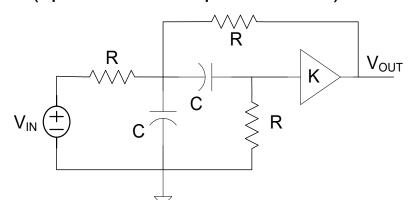
6 degrees of freedom (effectively 5 because dimensionless)

Denote as a +KRC filter

$$\omega_0 = ?$$
 BW = ?

Lots of flexibility (6 DOF but complicated expressions for ω_0 and Q)

Example 2 (special case of previous ckt):



Equal R, Equal C Realization

$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{RC} \frac{s}{s^2 + s \left(\frac{4 - K}{RC}\right) + \frac{2}{(RC)^2}}$$

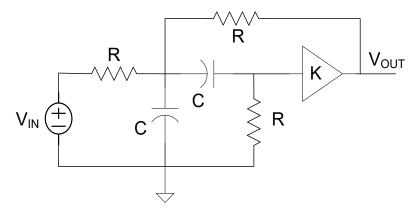
3 degrees of freedom (effectively 2 because dimensionless)

$$\omega_0 = ?$$

$$Q = ?$$

$$BW = ?$$

Example 2 (special case of previous circuit):



Equal R, Equal C Realization

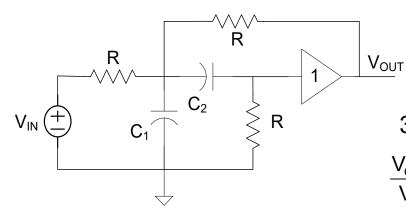
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{RC} \frac{s}{s^2 + s(\frac{4-K}{RC}) + \frac{2}{(RC)^2}}$$

$$\omega_0 = \frac{\sqrt{2}}{RC}$$
 $Q = \frac{\sqrt{2}}{4-K}$ $BW = \frac{4-K}{RC}$

3 degrees of freedom (effectively 2 since dimensionless)

- Can satisfy arbitrary 2nd=order BP constraints within a gain factor with this circuit
- · Very simple circuit structure
- Independent control of ω_0 and Q but requires tuning more than one component
- Can actually move poles in RHP by making K >4

Example 2 (another special case of previous circuit):



Unity Gain, Equal R

3 degrees of freedom (effectively 2 since dimensionless)

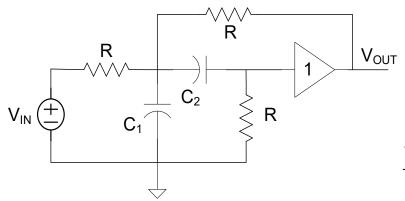
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{R C_1} \frac{s}{s^2 + s \left(\left[\frac{1}{R} \right] \left(\frac{2}{C_1} + \frac{1}{C_2} \right) \right) + \frac{2}{R^2 C_1 C_2}}$$

$$\omega_0 = ?$$

$$Q = ?$$

$$BW = ?$$

Example 2 (another special case of previous circuit):



Unity Gain, Equal R

$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{R C_1} \frac{s}{s^2 + s \left(\left[\frac{1}{R} \right] \left(\frac{2}{C_1} + \frac{1}{C_2} \right) \right) + \frac{2}{R^2 C_1 C_2}}$$

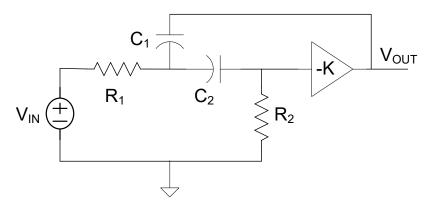
$$\omega_0 = \frac{\sqrt{2}}{R\sqrt{C_1C_2}}$$

$$\omega_0 = \frac{\sqrt{2}}{R\sqrt{C_1C_2}} \qquad \qquad Q = \sqrt{2}\sqrt{\frac{C_2}{C_1}} + \frac{1}{\sqrt{2}}\sqrt{\frac{C_1}{C_2}} \qquad \qquad BW = \left[\frac{1}{R}\right]\left(\frac{2}{C_1} + \frac{1}{C_2}\right)$$

$$BW = \left[\frac{1}{R}\right] \left(\frac{2}{C_1} + \frac{1}{C_2}\right)$$

Can't trim this filter with resistor

Example 3:



$$\frac{V_{\text{OUT}}}{V_{\text{IN}}} = T(s) = -\frac{K}{(1+K)R_1C_1} \frac{s}{s^2 + s \left(\left[\frac{1}{R_1C_1} + \frac{1}{R_2C_2} + \frac{1}{R_2C_1} \right] \frac{1}{(1+K)} \right) + \frac{1}{(1+K)R_1R_2C_1C_2}$$

Second-order Bandpass Filter

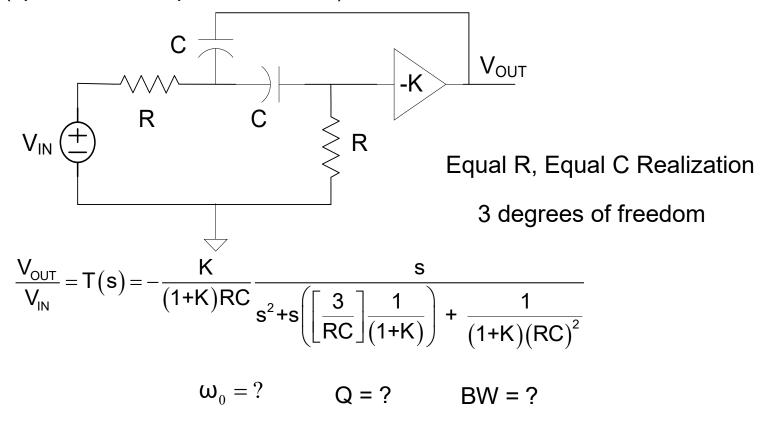
5 degrees of freedom (4 effective since dimensionless)

Denote as a -KRC filter

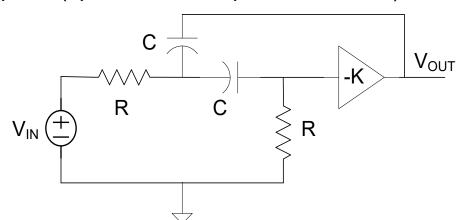
$$\omega_0 = ?$$

$$Q = ?$$

Example 3 (special case of previous circuit):



Example 3 (special case of previous circuit):



Equal R, Equal C Realization

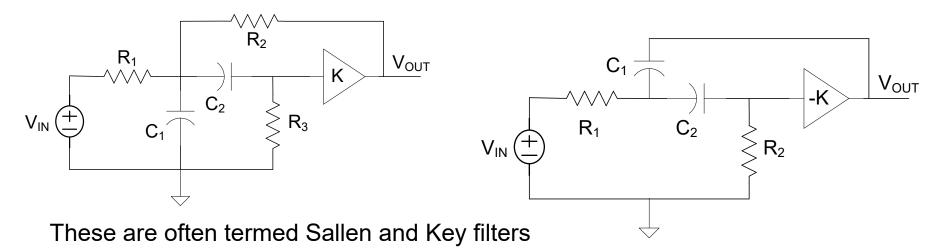
$$\frac{V_{\text{OUT}}}{V_{\text{IN}}} = T(s) = -\frac{K}{(1+K)RC} \frac{s}{s^2 + s \left(\left[\frac{3}{RC} \right] \frac{1}{(1+K)} \right) + \frac{1}{(1+K)(RC)^2}}$$

$$\omega_0 = \frac{1}{RC\sqrt{1+K}}$$
 Q = $\frac{\sqrt{1+K}}{3}$ BW = $\frac{3}{RC(1+K)}$

3 degrees of freedom (2 effective since dimensionless)

- Can satisfy arbitrary 2nd=order BP constraints within a gain factor with this circuit
- Very simple circuit structure
- Simple design process (sequential but not independent control of ω_0 and Q, requires tuning of more than 1 component if Rs used)

Observation:



Sallen and Key introduced a host of filter structures

Sallen and Key structures comprised of summers, RC network, and finite gain amplifiers

These filters were really ahead of their time and appeared long before practical implementations were available

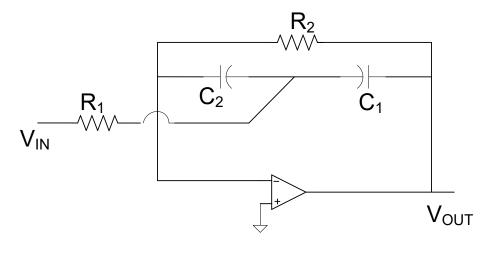
IRE TRANSACTIONS—CIRCUIT THEORY

March 1955

A Practical Method of Designing RC Active Filters*

R. P. SALLEN† AND E. L. KEY†

Example 4:



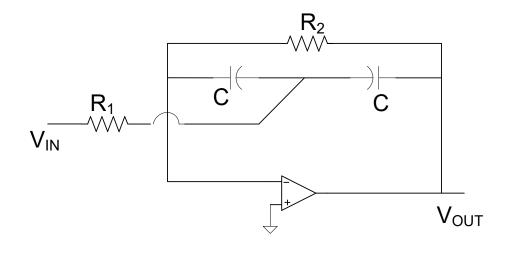
$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_1 C_1} \frac{s}{s^2 + s \left(\frac{1}{R_2} \left[\frac{1}{C_1} + \frac{1}{C_2}\right]\right) + \frac{1}{R_1 R_2 C_1 C_2}}$$

Second-order Bandpass Filter

4 degrees of freedom (3 effective since dimensionless)

Denote as a bridged T feedback structure

Example 4 (special case of previous circuit):



Equal C implementation

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_1 C} \frac{s}{s^2 + s \left(\frac{2}{CR_2}\right) + \frac{1}{R_1 R_2 C^2}}$$

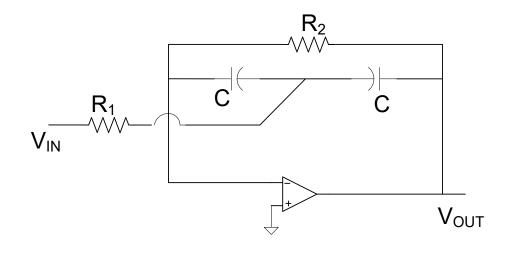
3 degrees of freedom (2 effective since dimensionless)

$$\omega_0 = ?$$

$$Q = ?$$

$$BW = ?$$

Example 4 (special case of previous circuit):



Equal C implementation

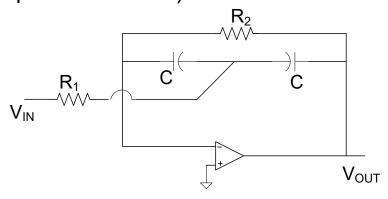
$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_1 C} \frac{s}{s^2 + s \left(\frac{2}{CR_2}\right) + \frac{1}{R_1 R_2 C^2}}$$

$$\omega_0 = \frac{1}{C\sqrt{R_1R_2}}$$
 $Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}}$ $BW = \frac{2}{R_2C}$

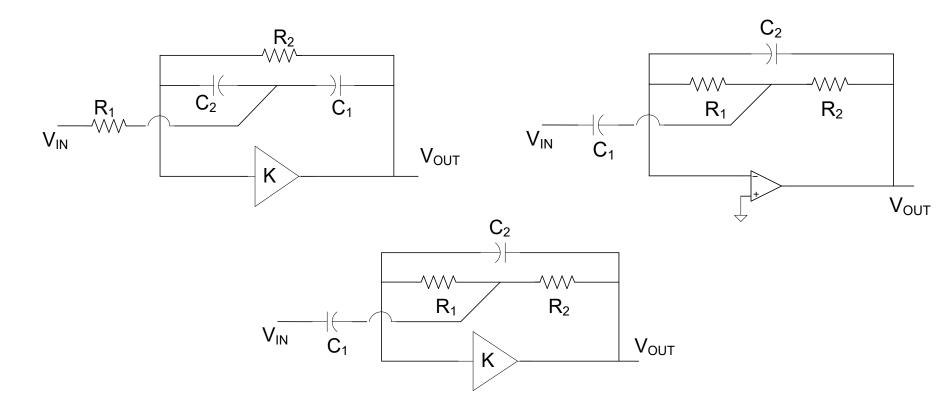
Simple circuit structure

More tedious design/calibration process for ω_0 and Q (iterative if C is fixed) Resistor ratio is $4Q^2$

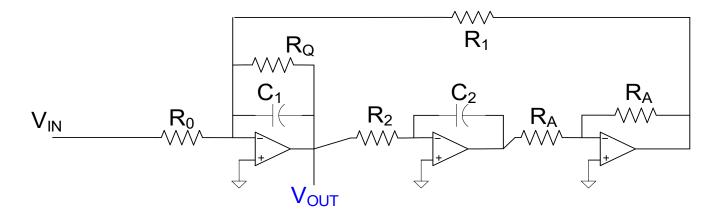
Example 4 (special case of previous circuit):



Some variants of the bridged-T feedback structure



Example 5:



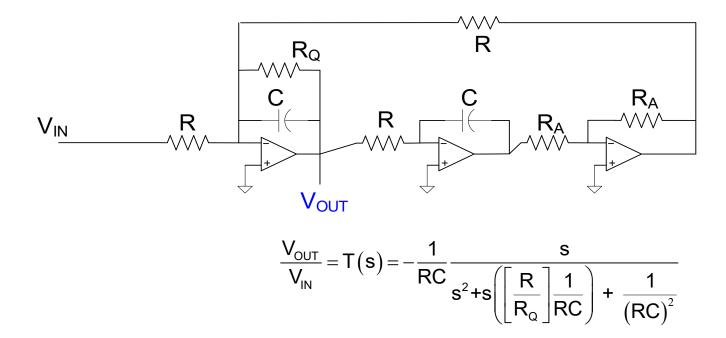
$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_0 C_2} \frac{s}{s^2 + s \left(\frac{1}{R_0 C_2}\right) + \frac{1}{R_1 R_2 C_1 C_2}}$$

Second-order Bandpass Filter

8 degrees of freedom (effectively 7 since dimensionless)

Denote as a two-integrator-loop structure

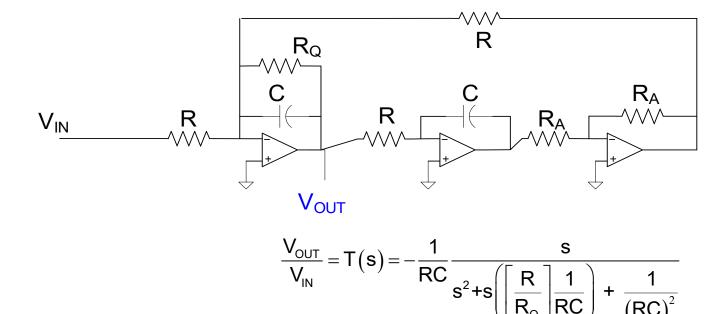
Often termed the Tow-Thomas Biquad



Equal R Equal C (except R_Q)

3 degrees of freedom (effectively 2 since dimensionless)

$$\omega_0 = ?$$
 BW = ?



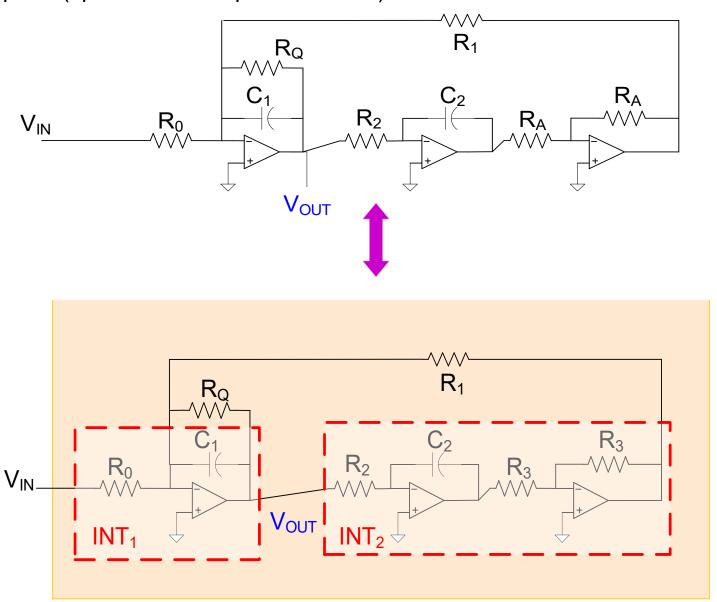
Equal R Equal C (except R_0)

3 degrees of freedom (effectively 2 since dimensionless)

$$\omega_0 = \frac{1}{RC}$$
 $Q = \frac{R_Q}{R}$ $BW = \left[\frac{R}{R_Q}\right] \frac{1}{RC}$

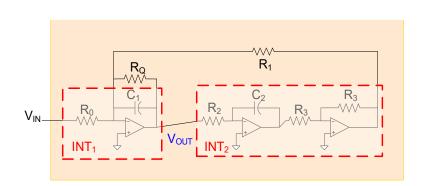
Simple design process (sequential but not independent control of ω_0 and Q with R's, requires more tuning more than one R if C's fixed)

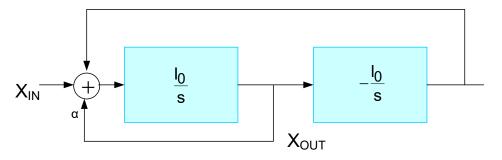
Modest component spread even for large Q



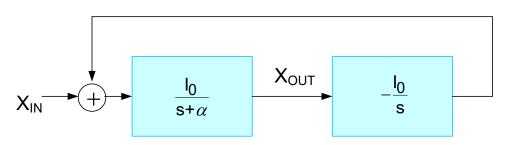
Two Integrator Loop Representation

Two Integrator Loop Representation

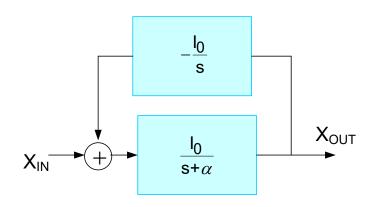




Inverting and Noninverting Integrator Loop

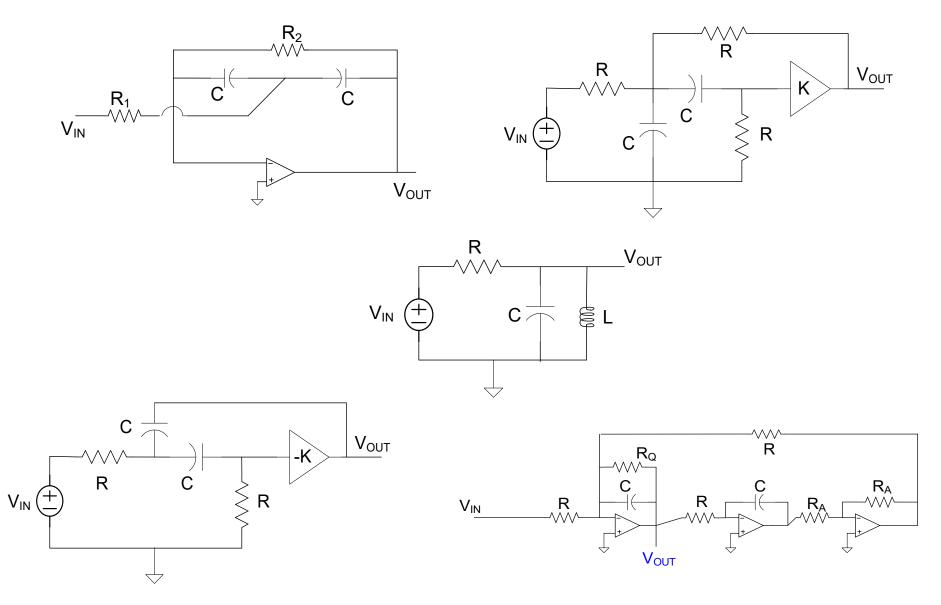


Integrator and Lossy Integrator Loop



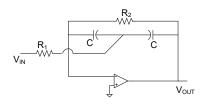
Integrator and Lossy Integrator Loop

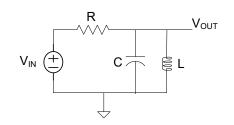
How does the performance of these bandpass filters compare?

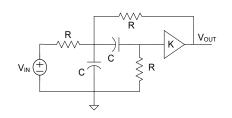


Ideally, all give same performance (within a gain factor)

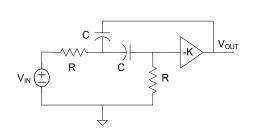
How does the performance of these bandpass filters compare?

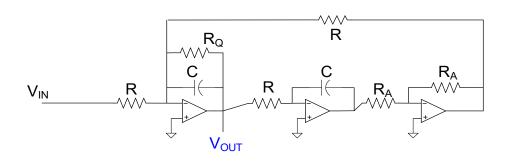






- Component Spread
- Number of Op Amps
- Is the performance strongly dependent upon how DOF are used?
- Ease of tunability/calibration (but practical structures often are not calibrated)
- Total capacitance or total resistance
- Power Dissipation
- Sensitivity
- Effects of Op Amps

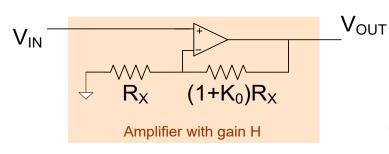


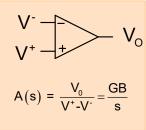


Consider effects of Op Amp on +KRC Bandpass with Equal R, Equal C

$$\omega_0 = \frac{\sqrt{2}}{RC} \qquad Q = \frac{\sqrt{2}}{4-K}$$

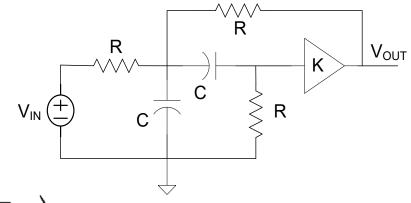
Assume K realized with standard Op Amp Circuit





$$K(s) = \frac{K_0}{1 + \frac{K_0}{GB}s}$$

- Significant shift in peak frequency
- BW does not change very much
- Some drop in gain at peak frequency



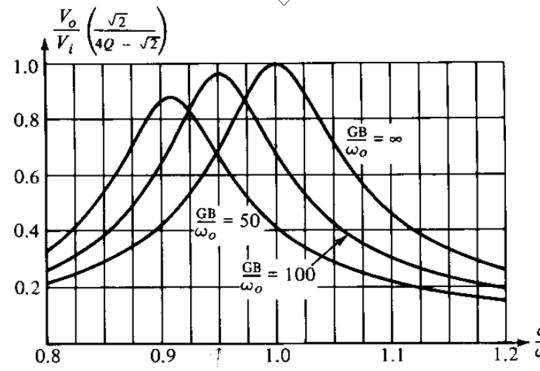
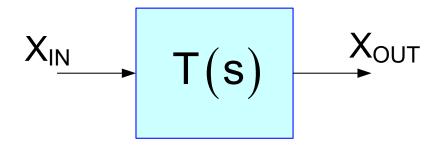


Fig. 11-4 Effect of GB on the magnitude curve for Q = 10

Consider 2nd Order Lowpass Biquads



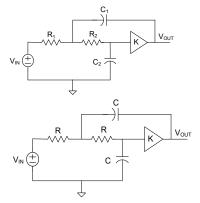
$$|T(s)| = H \frac{\omega_0^2}{s^2 + s(\frac{\omega_0}{Q}) + \omega_0^2}$$

$$BW = \omega_{B} - \omega_{A} \neq \frac{\omega_{0}}{Q}$$
$$\omega_{PEAK} \neq \omega_{0}$$

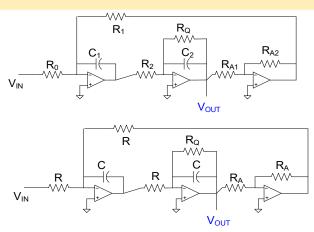
Consider 2nd Order Lowpass Biquads

$$|T(s)| = H \frac{\omega_0^2}{s^2 + s(\frac{\omega_0}{Q}) + \omega_0^2}$$

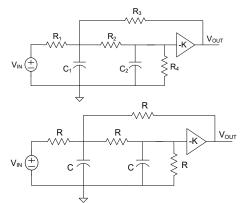
Four basic structures that ideally implement the same transfer function



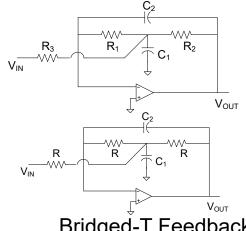
Sallen and Key +KRC



Two Integrator Loop

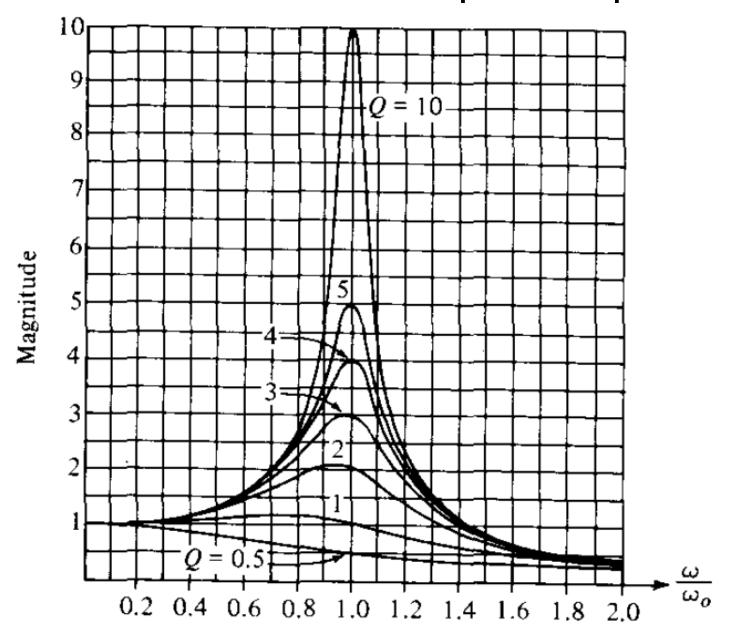


Sallen and Key -KRC

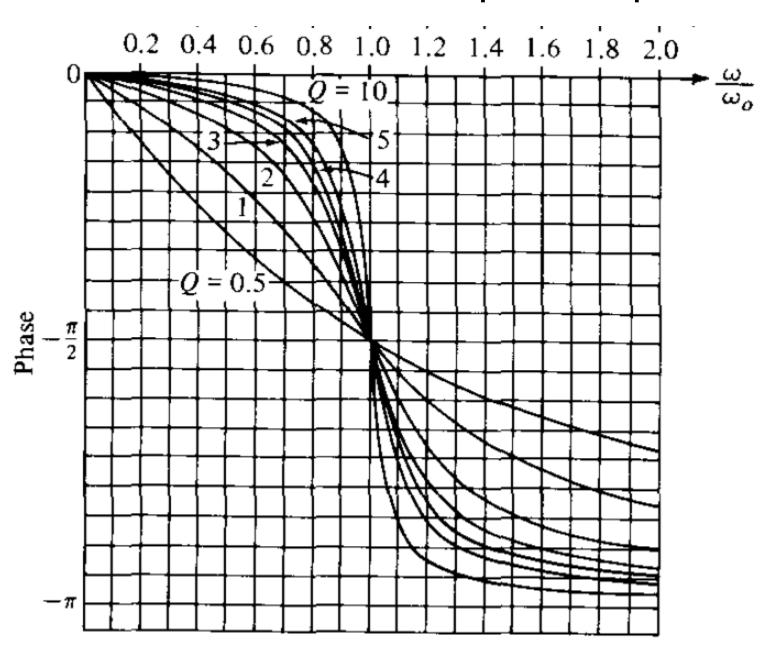


Bridged-T Feedback

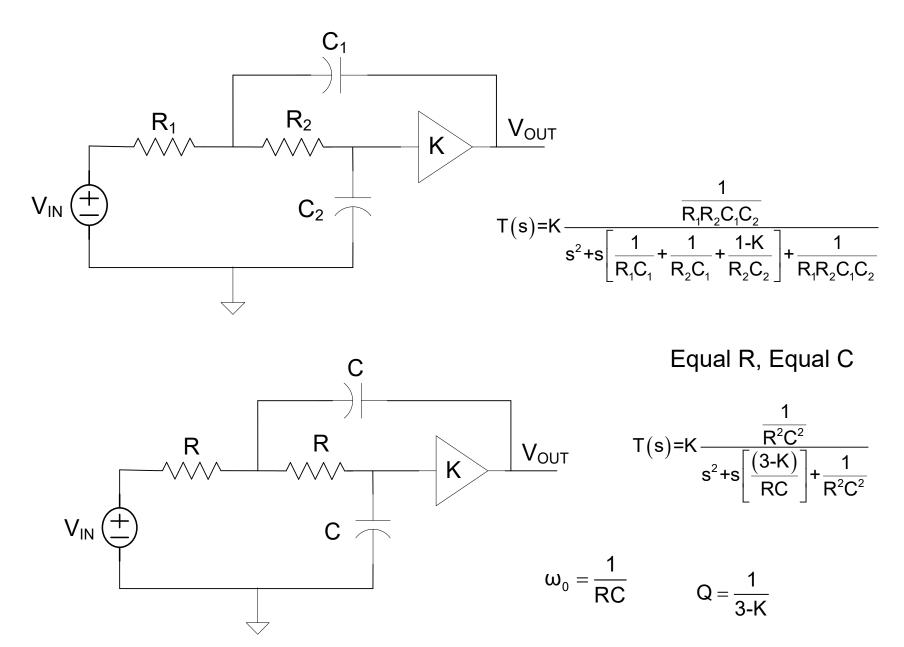
Consider 2nd Order Lowpass Biquads



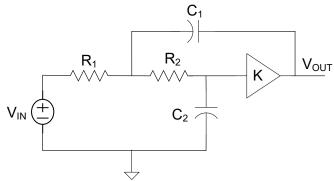
Consider 2nd Order Lowpass Biquads

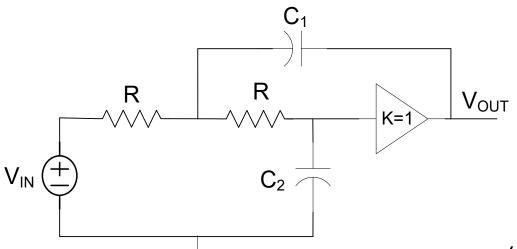


Example: 2nd Order +KRC Lowpass



Example: 2nd Order +KRC Lowpass



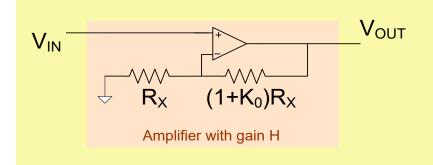


Equal R, K=1

$$T(s) = K \frac{\frac{1}{R^2 C_1 C_2}}{s^2 + s \left[\frac{2}{RC_1}\right] + \frac{1}{R^2 C_1 C_2}}$$

$$\omega_0 = \frac{1}{R\sqrt{C_1C_2}}$$

$$Q = \frac{1}{2} \sqrt{\frac{C_1}{C_2}}$$

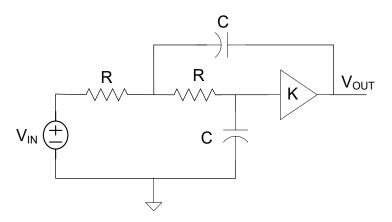


$$V^{-} \longrightarrow V_{O}$$

$$A(s) = \frac{V_{0}}{V^{+} - V^{-}} = \frac{GB}{s}$$

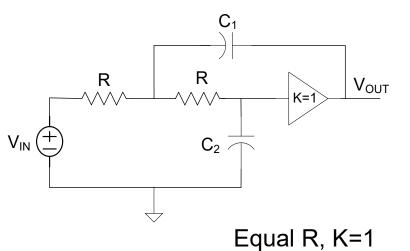
$$K(s) = \frac{K_0}{1 + \frac{K_0}{GB}s}$$

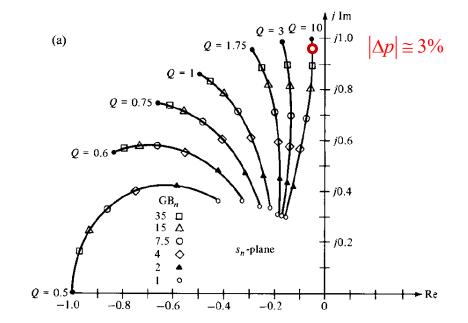
Example: 2nd Order +KRC Lowpass

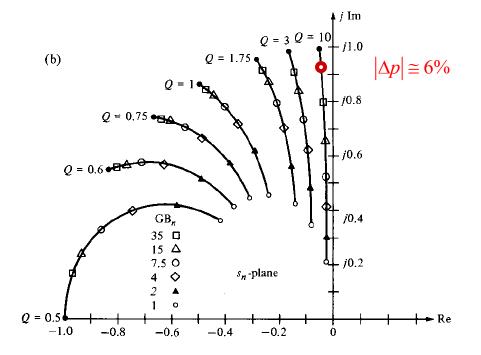


Equal R, Equal C

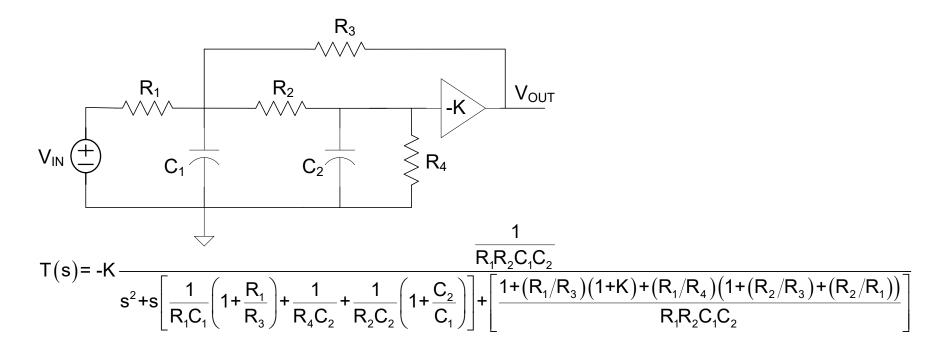
consider
$$\bullet \quad \longleftrightarrow \quad GB_n = \frac{GB}{\omega_0} = 100$$

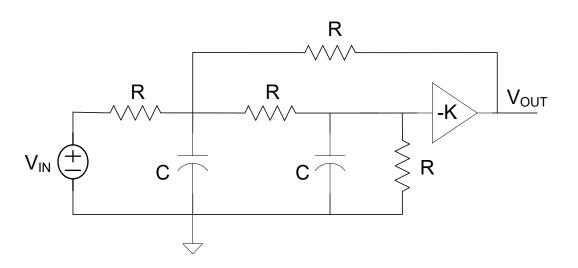






Example: 2nd Order -KRC Lowpass



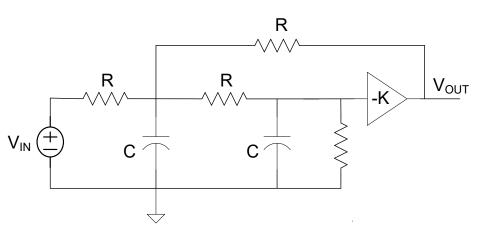


Equal R, Equal C

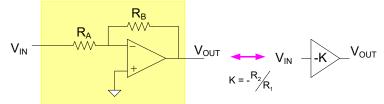
$$T(s) = -K \frac{\frac{1}{R^2C^2}}{s^2 + s\left[\frac{5}{RC}\right] + \left[\frac{5 + K}{R^2C^2}\right]}$$

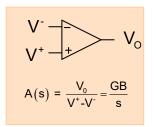
$$\omega_0 = \frac{\sqrt{5+K}}{RC} \qquad Q = \frac{\sqrt{5+K}}{5}$$

Example: 2nd Order -KRC Lowpass

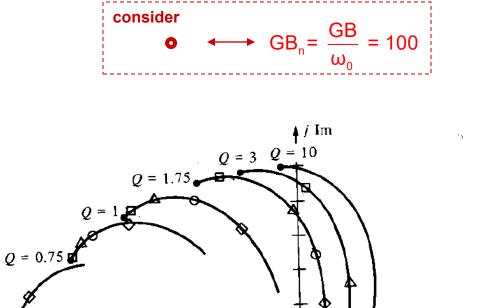


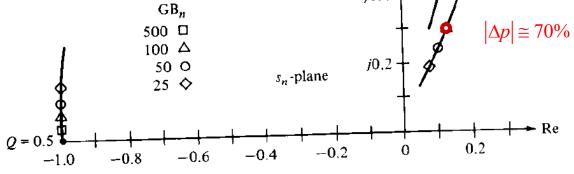
$$\omega_0 = \frac{\sqrt{5+K}}{RC}$$
 $Q = \frac{\sqrt{5+K}}{5}$





$$K(s) = -\frac{K_0}{1 + \frac{(1 + K_0)s}{GB}}$$

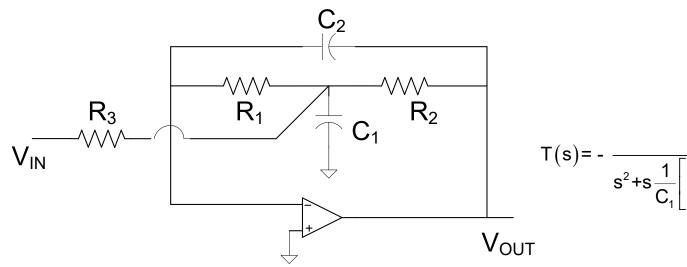




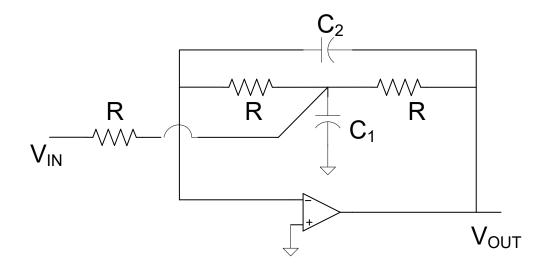
Poles "move" towards RHP as GB degrades Even very large values of GB will cause instability

Q = 0.6

Example: 2nd Bridged-T FB Lowpass



$$T(s) = -\frac{\frac{1}{R_2 R_3 C_1 C_2}}{s^2 + s \frac{1}{C_1} \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right] + \frac{1}{R_1 R_2 C_1 C_2}}$$

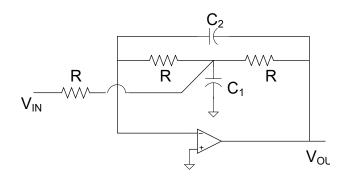


Equal R

$$T(s) = -\frac{\frac{1}{R^2 C_1 C_2}}{s^2 + s \left(\frac{3}{RC_1}\right) + \frac{1}{R^2 C_1 C_2}}$$

$$\omega_0 = \frac{1}{R\sqrt{C_1C_2}} \qquad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

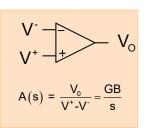
Example: 2nd Bridged-T FB Lowpass

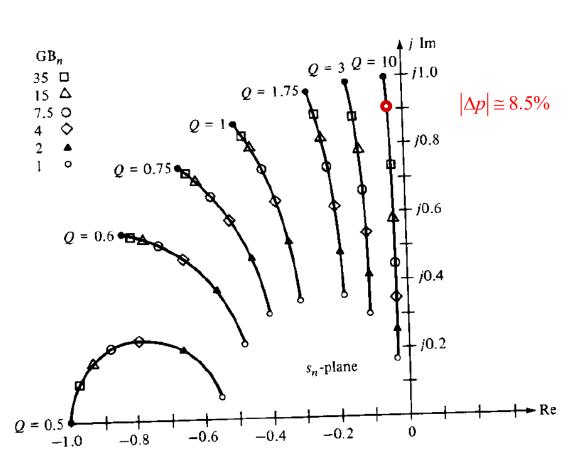


$$\omega_0 = \frac{1}{R\sqrt{C_1C_2}} \qquad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

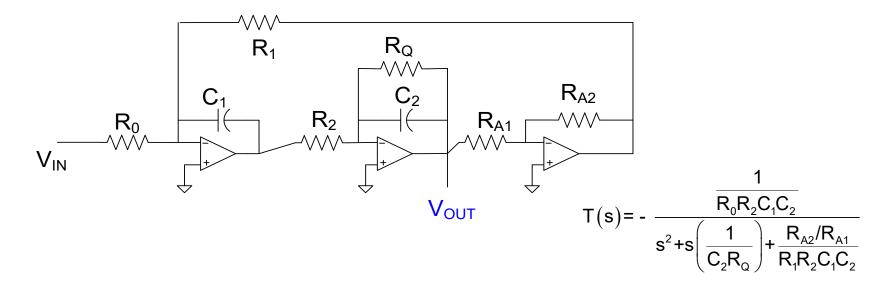
consider

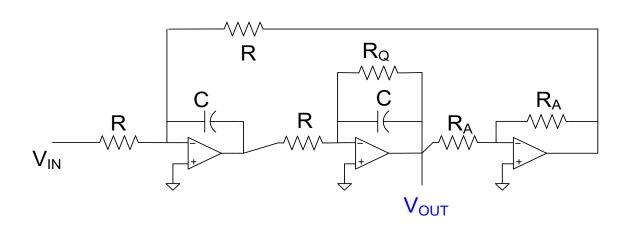
$$\bullet \qquad \longleftrightarrow \qquad GB_n = \frac{GB}{\omega_0} = 100$$





Example: 2nd Two-Integrator-Loop Lowpass



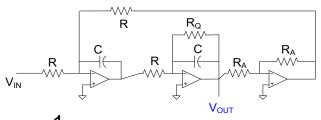


Equal R, Equal C (except R_Q)

$$T(s) = -\frac{\frac{1}{R^2C^2}}{s^2 + s\left(\frac{1}{CR_Q}\right) + \frac{1}{R^2C^2}}$$

$$\omega_0 = \frac{1}{RC} \qquad Q = \frac{R_Q}{R}$$

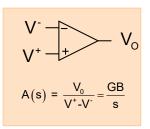
Example: 2nd Two-Integrator-Loop Lowpass

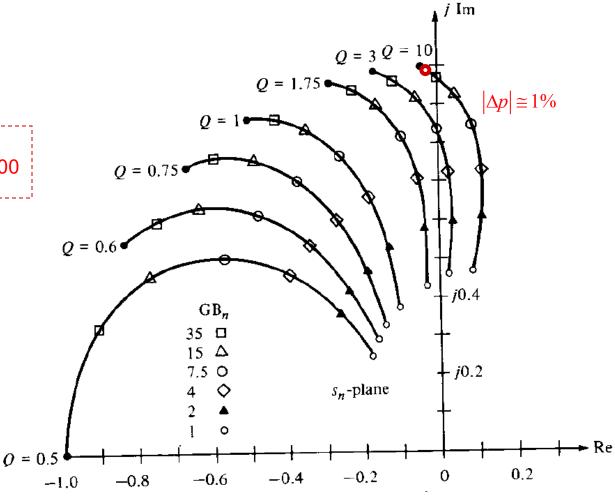


$$\omega_0 = \frac{1}{RC}$$

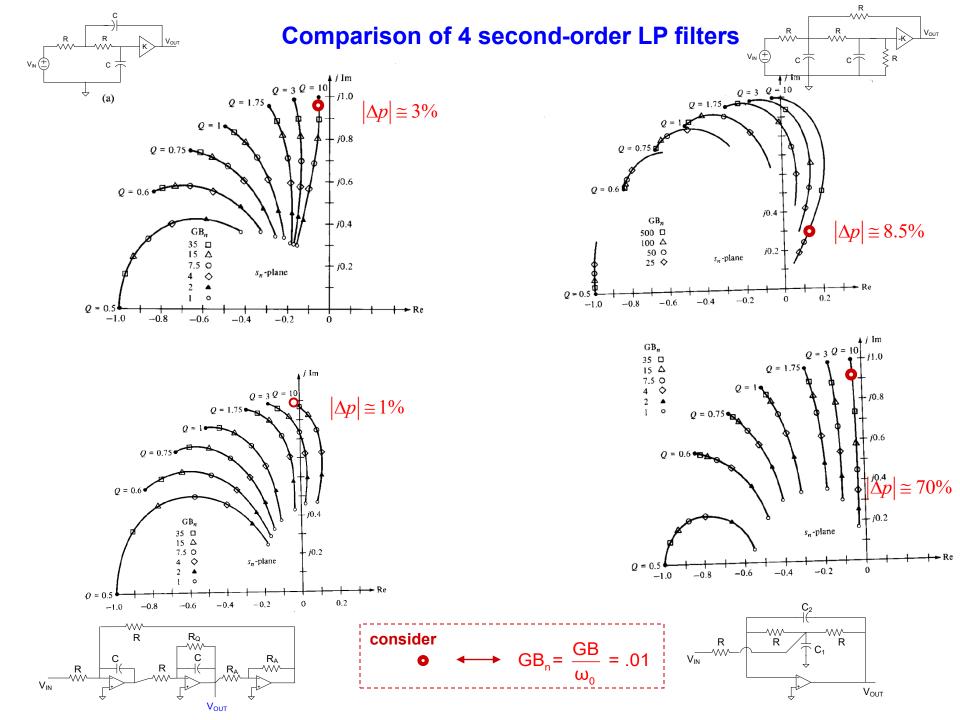
$$Q = \frac{R_Q}{R}$$







Poles "move" towards RHP as GB degrades



Some Observations

- Seemingly similar structures have dramatically different sensitivity to frequency response of the Op Amp
- Critical to have enough GB if filter is to perform as desired
- Performance strongly affected by both magnitude and direction of pole movement
- Some structures appear to be totally impractical at least for larger Q
- Different use of the Degrees of Freedom produces significantly different results

Sensitivity analysis is useful for analytical characterization of the performance of a filter



Stay Safe and Stay Healthy!

End of Lecture 17